

الامتحان مكون من (5) أسئلة مكتوبة في صفحة واحدة و المطلوب الإجابة على كل الأسئلة.

Time: 3 Hours

(1)(a) Test the series $\sum_{n=1}^{\infty} \frac{n!}{2n+1}$

(b) Find the interval of convergence of the series: $\sum_{n=1}^{\infty} \frac{3}{2n+1} (3x-2)^n$

(c) Find the extrema of the function $f(x,y) = 4x - 3y - x^2 + y^3$

(2)(a) Find the tangent plane of the surface $4x^2 + 9y^2 = z$ at $(0, -1, 9)$

(b) Verify Euler's theorem for the the function: $f(x,y,z) = \frac{x^3}{3x+2z} + y^2$

(c) If $\phi = xyz$, $\bar{U} = xyi + y^2j + 3zk$. Find $\nabla \cdot (\phi \bar{U})$, $\nabla \times (\phi \bar{U})$

(3) Solve the following differential equations:

(a) $(1+y)\cos x dx + y(1+\sin x)dy = 0$ (b) $y'' + 2y = 2 + e^{-3x}$

(c) $y'' - 3y' - 4y = 4 + e^{4x}$ (d) $y'' - 9y = x^2 + 2\sin^2 2x$

(4)(a) Find the series solution of the equation $(1+x^2)y'' - y = 0$

(b) Find the surface area of the surface $x^2 + y^2 + z = 1, z \geq 0$

(5) Evaluate the following integrals:

(a) $\int_0^{2x} \int_0^{10} (x+2y) dy dx$

(b) $\iint_D \sqrt{x^2 + y^2} dx dy$, D is the region inside the circle $x^2 + y^2 = 4$

(c) $\int_{(0,0)}^{(1,1)} (4x+2y)dx + (2x-y^2)dy$, through the curves:

(i) $y = x^2$

(ii) $x = y^2$

(iii) $y = x$

Good Luck

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